Student Number: ______________________

Teacher: ______________________

Class: ______________________

FORT STREET HIGH SCHOOL

2018

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics

Time allowed: 3 hours
(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
	Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-10
H3	Manipulates algebraic expressions involving logarithmic and exponential functions	11
H2	Constructs arguments to prove and justify results	13
H5, H6, H8	Applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems. Uses the derivative to determine the features of the graph of a function	12, 14, 15
H4, H9	Expresses practical problems in mathematical terms based on simple given models. Communicates using mathematical language, notation, diagrams and graphs	16

Total Marks 100

Section I 10 marks
Multiple Choice, attempt all questions,
Allow about 15 minutes for this section

Section II 90 Marks
Attempt Questions 11-16,
Allow about 2 hours 45 minutes for this section

General Instructions:

- Questions 11-16 are to be started in a new booklet
- The marks allocated for each question are indicated
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

Section I	Total 10	Marks
Q1-Q10		
Section II	Total 90	Marks
Q11	/15	
Q12	/15	
Q13	/15	
Q14	/15	
Q15	/15	
Q16	/15	
	Percent	

Section I

10 marks
Attempt Questions 1 to 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 to 10

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1. What is the locus of a set of points that is equidistant from a fixed point and a fixed line?

(A) a parabola **(B)** a hyperbola **(C)** a circle **(D)** a straight line

2. Which one of the following quadratic equations has two distinct real roots?

(A) $y = x^2 - 4x + 4$ **(B)** $y = x^2 + 4x + 4$

(C) $y = x^2 - 4x - 4$ **(D)** $y = x^2 + 4$

3. The solutions of $\sqrt{3}\tan x = -1$ for $0 \le x \le 2\pi$ are?

(A) $\frac{2\pi}{3}$ *and* $\frac{4\pi}{3}$ **(B)** $\frac{2\pi}{3}$ *and* $\frac{5\pi}{3}$ **(C)** $\frac{5\pi}{6}$ *and* $\frac{7\pi}{6}$ **(D)** $\frac{5\pi}{6}$ *and* $\frac{11\pi}{6}$

4. Find the limiting sum of the geometric series $\frac{2}{3} - \frac{2}{15} + \frac{2}{75} - \frac{2}{375} + \ldots$

(A) $\frac{3}{5}$ **(B)** 0 **(C)** $\frac{12}{15}$ **(D)** $\frac{5}{9}$

5. Which of the following conditions for $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$ describes the slowing growth of a variable B?

(A) $\frac{dP}{dt} > 0$ and $\frac{d^2P}{dt^2} > 0$ **(B)** $\frac{dP}{dt} < 0$ and $\frac{d^2P}{dt^2} < 0$

(C) $\frac{dP}{dt} > 0$ and $\frac{d^2P}{dt^2} < 0$ **(D)** $\frac{dP}{dt} < 0$ and $\frac{d^2P}{dt^2} > 0$

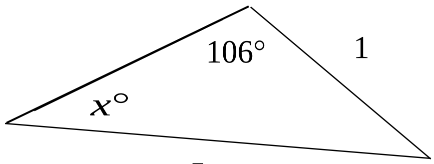
6. The quadratic equation $x^2+4x-1=0$ has roots α and β.
What is the value of $\alpha\beta+(\alpha+\beta)$?

(A) 5 **(B)** 3 **(C)** -5 **(D)** -3

7. If $\ln a = \ln b + \ln c$, then which of these is true?

(A) $a = bc$ **(B)** $a = b + c$ **(C)** $\ln a = bc$ **(D)** $a = \frac{b}{c}$

8. Which calculation gives the value of x in the diagram below?



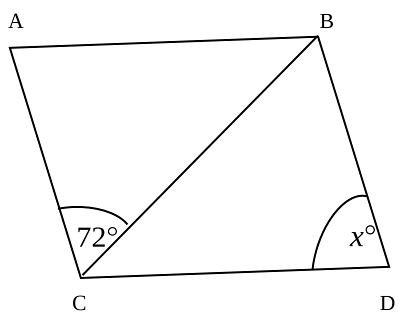
(A) $x=\sin^{-1}\left(\frac{\sin 106°}{5}\right)$ **(B)** $x=\cos^{-1}\left(\frac{\sin 106°}{5}\right)$

(C) $x=\sin^{-1}(5\sin 106°)$ **(D)** $x=\sin^{-1}\left(\frac{1}{5}\right)$

9. A bag contains red and green lollies in the ratio of 7 : 2 . If a lolly is selected at random, what is the probability that it is a green lolly?

(A) $\frac{1}{7}$ **(B)** $\frac{7}{9}$ **(C)** $\frac{2}{7}$ **(D)** $\frac{2}{9}$

10. The quadrilateral ABDC below is a rhombus.
What is the value of the angle $x°$ marked on the diagram?



(A) 18° **(B)** 36° **(C)** 46° **(D)** 72°

End of Section I

Section II

90 marks
Attempt Questions 11 to 16
Allow about 2 hours 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.
In Questions11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Start a new writing booklet.**

(a) Factorise $9x^2 - 16$. **1**

(b) Rationalise the denominator of $\dfrac{1}{2+\sqrt{5}}$. **2**

(c) Solve $|x-1| \geq 4$. **2**

(d) Differentiate $\left(3+e^{2x}\right)^5$. **2**

(e) Find $\displaystyle\int \frac{6x^2}{x^3+1}\,dx$. **2**

(f) Solve $\sin^2 x + 2\cos x = 1$ for $0 \leq x \leq 2\pi$. **3**

(g) Sketch the region defined by $(x-2)^2 + (y-3)^2 < 4$. **3**

End of Question 11

Question 12 (15 marks) **Start a new writing booklet.**

(a) Solve $5^x = 4$ correct to 1 decimal place. **2**

(b) Find the equation of the tangent to the curve $y = e^x$ at the point $(1, e)$. **2**

(c) Evaluate the definite integral **2**

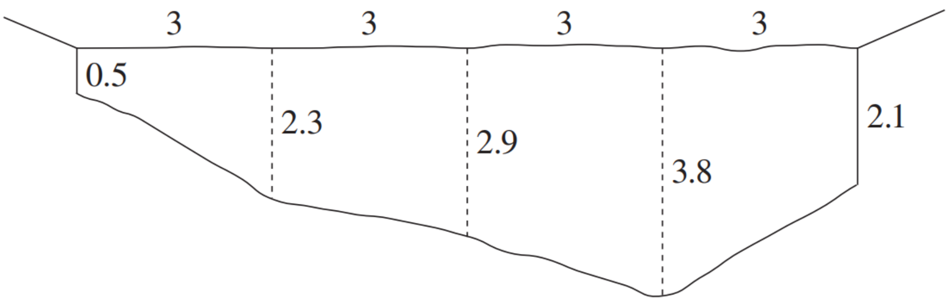
$$\int_{-1}^{3} (6x - 7)\,dx\,.$$

(d) The gradient of a curve is given by $\frac{dy}{dx} = 9x^2 - 2x + 1$. The curve passes through the point $(-1, -4)$. What is the equation of the curve? **3**

(e) A sum of \$20 000 is invested at a fixed rate of interest, compounded annually. After 5 years the principal has grown to \$28 567. **3**

Find the annual rate of interest as a percentage correct to one decimal place.

(f) Below is a diagram showing the cross-section of a creek, with depths of the creek given in metres, at 3 metre intervals. The creek is 12 metres in width.



(i) Use Simpson's rule with five depth measurements to calculate the approximate area of the cross-section. **2**

(ii) If the water flows through this section of the creek at $0 \cdot 5\ m^{-1}$. Calculate the approximate volume of water that flows past this section in ten seconds. **1**

End of Question 12

Question 13 (15 marks) **Start a new writing booklet.**

(a) Evaluate **2**

$$\int_0^{\frac{\pi}{2}} \sec^2 \frac{x}{2}\, dx .$$

(b) **(i)** Show that the perpendicular distance from the point $(1,-4)$ to the line $4x-3y+14=0$ is 6 units. **1**

(ii) Find the centre and radius of the circle $x^2-2x+y^2+8y=8$. **2**

(iii) Explain why the line $4x-3y+14=0$ will never intersect the circle $x^2-2x+y^2+8y=8$. **1**

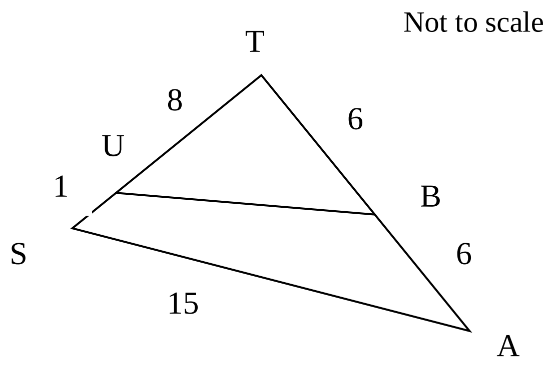
(c) A particle is moving in a straight line. At time t seconds its displacement is x metres from the fixed point O on the line and its velocity is given by $v=3t^2-2t-1$. Initially the particle is 1 metre to the right of O.

(i) Show that the particle is at rest after 1 second. **1**

(ii) Find the displacement x in terms of t. **2**

(iii) Find the distance travelled by the particle in the first 2 seconds. **2**

(d)



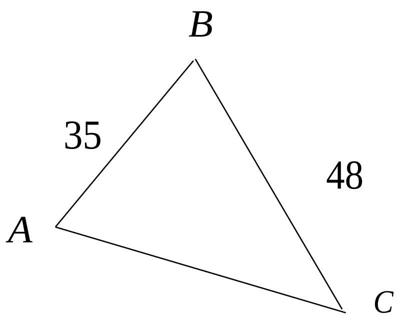
(i) Prove that ΔBUT is similar to ΔSAT. **2**

(ii) Hence, or otherwise, find the length of BU. **2**

End of Question 13

Question 14 (15 marks) **Start a new writing booklet.**

(a) The bearing of B from A is $036°T$ and the bearing of C from B is $156°T$.



Not to scale

Copy the diagram into your writing booklet

(i) Show that the value of $\angle ABC = 60°$. **2**

(ii) Find the distance AC . **2**

(iii) Find the bearing of A from C . **2**

(b)

(i) Sketch the curve $y = 3 + \cos 2x$ for $-\pi \le x \le \pi$. **3**

(ii) Find the exact value of the area under the curve $y = 3 + \cos 2x$ between the x - axis, $x = 0$ and $x = \dfrac{7\pi}{12}$. **2**

(c) Two players, in a game, take turns at drawing and then immediately replacing a marble from a bag. The bag contains 2 green and 3 red marbles.
Player A draws first.
For A to win he must draw a green marble.
For B to win he must draw a red marble.

Find the probability that:

(i) A wins on his first draw. **1**

(ii) B wins on his first draw. **1**

(ii) A wins in fewer than 4 of his turns. **2**

End of Question 14

Question 15 (15 marks) **Start a new writing booklet.**

(a)

(i) Differentiate $x^2 \ln x$. **1**

(ii) Hence, or otherwise, find $\int 3x\left(1+\ln x^2\right)\,dx$. **2**

(b) Consider the function $f\left(x\right)=3-3x^2-x^3$ in the domain $-3 \le x \le 2$.

(i) Find the stationary points and determine their nature. **3**

(ii) Find the point of inflexion. **2**

(iii) Draw a sketch of the curve $y=f\left(x\right)$ in the domain $-3 \le x \le 2$. **2**

(iv) What is the minimum value of the function in the given domain? **1**

(c) Jamie borrows \$35000 at 18% p.a. reducible interest. She plans to repay the loan in equal monthly instalments over 5 years. If A_n is the amount owing after n instalments and M is the amount paid in each instalment:

(i) Show that $A_2 = 35000 \times 1\cdot 015^2 - M\left(1\cdot 015+1\right)$ **1**

(ii) Find the amount she needs to pay for each instalment to pay off the loan. Answer to the nearest dollar. **3**

End of Question 15

Question 16 (15 marks) **Start a new writing booklet.**

(a) Find the exact value of the volume of the solid formed when the area bounded by the curve $y=3-x^2$, for $x \geq 0$, the $y-axis$ and the line $y=2$ is rotated about the $x-axis$. **4**

(b) The death rate of an endangered species on an island is given by

$$\frac{dP}{dt}=-kP,$$

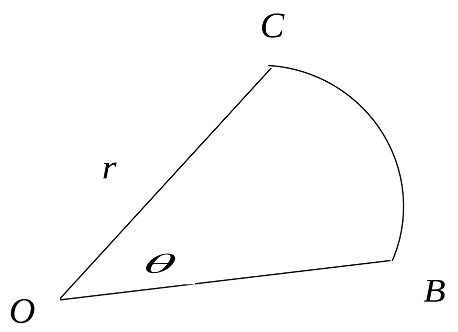
where P is the population of the species after t days and k is a constant.

(i) Show that $P=Ae^{-kt}$ is a solution to the equation, where A is a constant. **1**

(ii) Initially there were 2000 of the species on the island, after 300 days only 1000 were left. What is the population (to the nearest whole number) after 400 days? **2**

(iii) After how many days will the population drop below 400? **2**

(c) The diagram below shows a sector of a circle with centre O and radius $r\ cm$. The arc subtends an angle θ radians at O and the area of the sector is $8\ cm^2$.



(i) Find an expression for r in terms of θ. **1**

(ii) Show that the perimeter of the sector is given by $P=\frac{8}{\sqrt{\theta}}+4\sqrt{\theta}$. **1**

(iii) If $0 \leq \theta \leq \pi$, find the value of θ for a minimum perimeter. **4**

End of paper

Student Number: ____________________

Teacher: ____________________

Class: ____________________

FORT STREET HIGH SCHOOL

2018

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: SOLUTIONS

Mathematics

Time allowed: 3 hours
(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
	Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-10
H2, H3, H4, H5	Manipulates algebraic expressions to solve problems from topic areas such as geometry, co-ordinate geometry, quadratics, trigonometry, probability and logarithms	12, 14
H6, H7, H8	Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	11, 13, 15
H9	Synthesises mathematical solutions to harder problems and communicates them in appropriate form	16

Total Marks 100

Section I   10 marks
Multiple Choice, attempt all questions,
Allow about 15 minutes for this section
Section II   90 Marks
Attempt Questions 11-16,
Allow about 2 hours 45 minutes for this section

General Instructions:

- Questions 11-16 are to be started in a new booklet
- The marks allocated for each question are indicated
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

Section I	Total 10	Marks
Q1-Q10		
Section II	Total 90	Marks
Q11	/15	
Q12	/15	
Q13	/15	
Q14	/15	
Q15	/15	
Q16	/15	
	Percent	

Section I

10 marks
Attempt Questions 1 to 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 to 10

1. What is the locus of a set of points that is equidistant from a fixed point and a fixed line?

(A) **a parabola**	**(B)** a hyperbola	**(C)** a circle	**(D)** a straight line

2. Which of the following quadratic equations have two distinct real roots?

(A) $y = x^2 - 4x + 4$	**(B)** $y = x^2 + 4x + 4$
(C) $y = x^2 - 4x - 4$	**(D)** $y = x^2 + 4$

3. The solutions of $\sqrt{3}\tan x = -1$ for $0 \le x \le 2\pi$ are?

(A) $\frac{2\pi}{3}$ *and* $\frac{4\pi}{3}$	**(B)** $\frac{2\pi}{3}$ *and* $\frac{5\pi}{3}$	**(C)** $\frac{5\pi}{6}$ *and* $\frac{7\pi}{6}$	**(D)** $\frac{5\pi}{6}$ *and* $\frac{11\pi}{6}$

4. Find the limiting sum of the geometric series $\frac{2}{3} - \frac{2}{15} + \frac{2}{75} - \frac{2}{375} + \ldots$

(A) $\frac{3}{5}$	**(B)** 0	**(C)** $\frac{12}{15}$	**(D)** $\frac{5}{9}$

5. Which of the following conditions for $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$ describes the slowing growth of a variable B?

(A) $\frac{dP}{dt} > 0$ and $\frac{d^2P}{dt^2} > 0$	**(B)** $\frac{dP}{dt} < 0$ and $\frac{d^2P}{dt^2} < 0$
(C) $\frac{dP}{dt} > 0$ **and** $\frac{d^2P}{dt^2}$ **< 0**	**(D)** $\frac{dP}{dt} < 0$ and $\frac{d^2P}{dt^2} > 0$

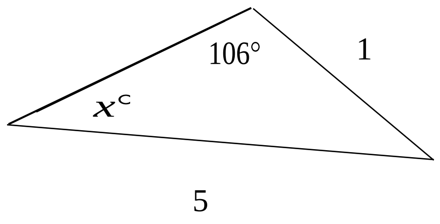
6. The quadratic equation $x^2 + 4x - 1 = 0$ has roots α and β.
What is the value of $\alpha\beta + (\alpha + \beta)$?

(A) 5 | **(B)** 3 | **(C)** -5 | **(D)** -3

7. If $\ln a = \ln b + \ln c$, then which of these is true?

(A) $a = bc$ | **(B)** $a = b + c$ | **(C)** $\ln a = bc$ | **(D)** $a = \frac{b}{c}$

8. Which calculation gives the value of x in the diagram below?



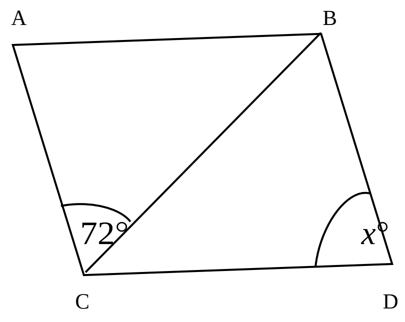
(A) $x = \sin^{-1}\left(\frac{\sin 106°}{5}\right)$ | **(B)** $x = \cos^{-1}\left(\frac{\sin 106°}{5}\right)$

(C) $x = \sin^{-1}(5\sin 106°)$ | **(D)** $x = \sin^{-1}\left(\frac{1}{5}\right)$

9. A bag contains red and green lollies in the ratio of 7 : 2 . If a lolly is selected at random, what is the probability that it is a green lolly?

(A) $\frac{1}{7}$ | **(B)** $\frac{7}{9}$ | **(C)** $\frac{2}{7}$ | **(D)** $\frac{2}{9}$

10. The quadrilateral ABDC below is a rhombus.
What is the value of the angle $x°$ marked on the diagram?



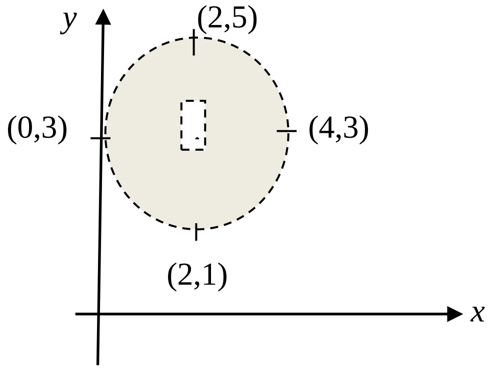
(A) 18° | **(B)** 36° | **(C)** 46° | **(D)** 72°

End of Section I

Section II

90 marks

Question 11 (15 marks)		**Teacher's Comments**
(a)	$9x^2 - 16 = (3x)^2 - (4)^2$ $= (3x-4)(3x+4)$ ❶	*Well Done*
(b)	$\frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{2-\sqrt{5}}{4-5}$ ❶ $= \frac{2-\sqrt{5}}{-1}$ $= \sqrt{5} - 2$ ❶	*Well done* *A few students did not simplify fully*
(c)	$\lvert x-1 \rvert \geq 4$ $x-1 \geq 4$, $x-1 \leq -4$ $x \geq 5$, ❶ $x \leq -3$ ❶  -3 5	*Mostly Well done* *Some students forgetting to reverse inequality sign when dividing/ multiplying by a negative* *Some answers showing students obviously did not test their results* $-3 \geq x \geq 5$ *is unacceptable*
(d)	$\frac{d}{dx}\left(3+e^{2x}\right)^5 = 5\left(3+e^{2x}\right)^4 . e^{2x} . 2$ ❶ $= 10e^{2x}\left(3+e^{2x}\right)^4$ ❶	*Well done* *Very few students failing to simplify the answer, need to recognise the mark allocated, requiring a simplified answer*
(e)	$\int \frac{6x^2}{x^3+1}\,dx = 2\int \frac{3x^2}{x^3+1}\,dx$ $= 2\ln\left(x^3+1\right) + c$ ❶ ❶	*Most students recognised the integral as a log.* *Care needed not to leave off the brackets and/ or the constant*
(f)	$\sin^2 x + 2\cos x = 1$ $1 - \cos^2 x + 2\cos x - 1 = 0$ *Let* $\cos x = u$ $1 - u^2 + 2u - 1 = 0$ $-u^2 + 2u = 0$ ❶ $u(2-u) = 0$ $\therefore\ u = 0, u = 2$ $\cos x = 0$, $\cos x = 2$ (*no solution*) ❶ $\therefore\ x = \frac{\pi}{2}, \frac{3\pi}{2}$ ❶	*Some students did not recognise the use of Trig. Identities.* *Marks lost for not answering the question i.e. including solutions outside* $0 \leq x \leq 2\pi$

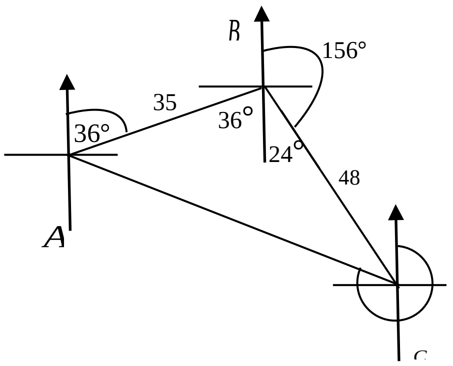
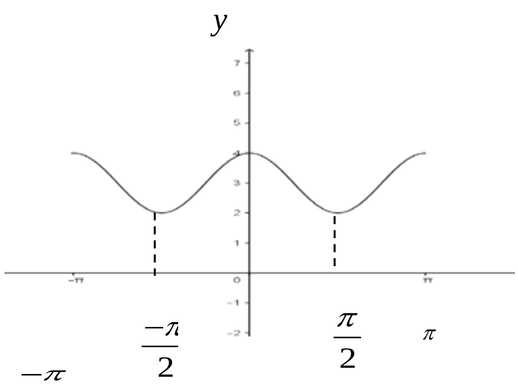
(g)	$(x-2)^2+(y-3)^2<4$ Region Shaded inside circle ❶ Broken line ❶ Centre (2, 3) , radius 2 ❶ 	*Many answers failed to show a broken line to exclude the points on the circle in region shaded.* *Many answers did not clearly label to show the features of the graph such as points to indicate the centre and radius of the circle.* *Some students did not recognise the equation as the graph of a circle.*
End of Question 11		

Question 12 (15 marks)		**Teacher's Comments**
(a)	$5^x=4$ $\ln 5^x=\ln 4$ ❶ $x\ln 5=\ln 4$ $x=\frac{\ln 4}{\ln 5}$ $=0\cdot 86135$ $\approx 0\cdot 9$ ❶ (correct to 1 decimal place)	**1 mark off for answering to 2dp (or other) instead of 1dp**
(b)	$\frac{dy}{dx}=e^x \quad at\ x=1$ $m=e^x=e^1=e$ ❶ Eqn. of tangent at (1, e) $y-e=e(x-1)$ $y=ex-e+e$ $y=ex$ ❶	**Good use of point gradient formula but** $m\neq e^x$ Sub $x=1$; ie: $m=e^x=e^1=e$ **General form is acceptable too:** $ex-y=0$
(c)	$\int_{-1}^{3}(6x-7)\,dx=3x^2-7x\Big]_{-1}^{3}$ ❶ $=\left[3(3)^2-7(3)\right]-\left[3(-1)^2-7(-1)\right]$ $=[6]-[10]$ $=-4$ ❶	**Watch your substitutions with negatives of negatives. Many people got +-10 instead of -4 due to substitution errors.**

(d)	$\int\left(9x^2-2x+1\right)\,dx=\frac{9x^3}{3}-\frac{2x^2}{2}+x+c$ $\therefore \quad y=3x^3-x^2+x+c$ ❶ $\sin ce\left(-1,-4\right) lies\ on\ the\ curve\ then$ $y=3\left(-1\right)^3-\left(-1\right)^2+\left(-1\right)+c$ $-4=-3-1-1+c$ ❶ $\therefore c=1\ \ and\ \ y=3x^3-x^2+x+1$ ❶	**Don't use the point gradient formula here. The point gradient formula is used for finding equations of straight lines, which are curves with constant gradients. This curve doesn't have a constant gradient, it's gradient is dependent on x.**
(e)	$A=20000\left(1+\frac{r}{100}\right)^5$ ❶ $28567=20000\left(1+\frac{r}{100}\right)^5$ $\left(\sqrt[5]{\frac{28567}{20000}}-1\right)\times 100=r$ ❶ $\therefore\ r=7\cdot 3907...$ $r\approx 7\cdot 4\%$ ❶	**Use compound interest, not simple interest.** **Convert your answer to a percentage, don't leave it as 0.074 or round to 0.1.** **Answer to 1dp as specified (not 2dp).**
(f)	(i) Area $\approx\frac{h}{3}\left(y_0+4y_1+2y_2+4y_3+\ 2y_2+y_5\right)$ $\approx\frac{3}{3}\left[0\cdot 5+\ 2\cdot 1+4\left(2\cdot 3+3\cdot 8\right)+2\left(2\cdot 9\right)\ \right]$ ❶ $\approx 32\cdot 8m^2$ ❶ **(ii)** Distance = speed x time $=0\cdot 5\times 10$ $=5m$ $Volume=32\cdot 8\times 5$ $=164m^3$ ❶	**Students made mistakes calculating h, which is the "strip" width, or smallest interval width, in this case: 3.** **It is also equal to (b – a) divided by the number of "strips" or smallest intervals, in this case: 12 / 4.** **ii) 164 m^3 not 5 and not 164L.** **In maths, volume is m^3 and capacity is L.** **P.S.: 1 m^3 = 1000L**
	End of Question 12	

Question 13 (15 marks)		Teacher's Comments
(a)	$$\int_0^{\frac{\pi}{2}} \sec^2 \frac{x}{2}\,dx = \left[\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]_0^{\frac{\pi}{2}}$$ $$= \left[2\tan\frac{x}{2}\right]_0^{\frac{\pi}{2}}$$ ❶ $$= 2\left[\tan\frac{\pi}{4} - \tan 0\right]$$ $$= 2[1-0]$$ $$= 2$$ ❶	*Generally well done.* *Some students placing the limits at the start of the brackets – e.g.* $\int_0^{\frac{\pi}{2}}\left[2\tan\frac{x}{2}\right]$ *which is not correct.*
(b)	**(i)** $$d = \left\|\frac{4(1)-3(-4)+14}{\sqrt{4^2+3^2}}\right\|$$ ❶ $$= \left\|\frac{4+12+14}{\sqrt{25}}\right\|$$ $$= \frac{\|30\|}{5}$$ $$= 6$$	*Generally well done.*
	(ii) $$x^2 - 2x + y^2 + 8y = 8$$ $$x^2 - 2x + 1 + y^2 + 8y + 16 = 8 + 1 + 16.$$ $$(x-1)^2 + (y+4)^2 = 25$$ ❶ Giving a circle centre (1, -4) , radius 5 ❶	*Some students had trouble completing the square correctly.*
	(iii) The line $4x - 3y + 14 = 0$ will never intersect the circle $x^2 - 2x + y^2 + 8y = 8$ since the distance of the line from the point (which is the centre of the circle) is 6 units and this is greater than the circles radius of 5 units. ❶	*Many students did not use the previous parts to easily answer this one. Only some were able to show* $\Delta < 0$ *to give the no intercept result. A few tried graphing, but without grid paper, accuracy cannot be guaranteed.*
(c)	**(i)** The particle is at rest (stationary) when $v = 0$ When $t = 1$, $$v = 3(1)^2 - 2(1) - 1$$ $$= 3 - 2 - 1$$ $$= 0$$ $\therefore$ the particle is at rest after 1 sec ❶	*Generally well done.*

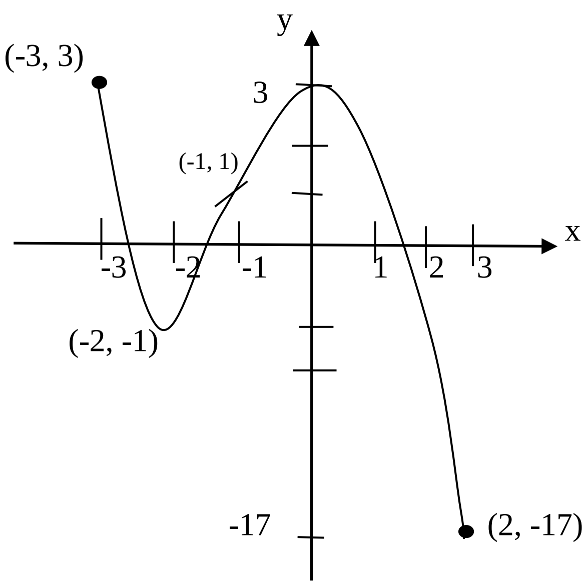
(c)	**(ii)** $x=\int\left(3t^2-2t-1\right)dt$ $=\frac{3}{3}t^3-\frac{2}{2}t^2-t+c$ $=t^3-t^2-t+c$ ❶ When $t=0$, $x=1$ $\therefore\ 1=0^3-0^2-0+c$ $c=1$ thus $x=t^3-t^2-t+1$ ❶ (displacement x in terms of t)	*Generally well done, but many abbreviated too many steps or made careless transcription errors.*
	(iii) $x=\left\|\int_0^1\left(3t^2+-2t-1\right)dt\right\|+\left\|\int_1^2\left(3t^2+-2t-1\right)dt\right\|$ $=\left\|\left[\left(t^3-t^2-1\right)\right]_0^1\right\|+\left\|\left[\left(t^3-t^2-1\right)\right]_1^2\right\|$ ❶ $=\left\|\left[(1-1-1)-0\right]\right\|+\left\|\left[(8-4-2)-(1-1-1)\right]\right\|$ $=\|-1\|+\|3\|$ $=4$ ❶ $\therefore$ the particle travels 4 metres in the first 2 seconds.	*Many students did not realize the significance of the stat. pt. at $x=1$ from the previous parts.* ***Note:*** *displacement $\neq$ distance travelled!!* *The most successful alternate solution was summing the distance travelled between the 0 and 1 second marks with that travelled between the 1 and 2 second marks.*
(d)	**(i)** $\angle T$ *is common* ❶ From $\Delta BUT : \Delta SAT$ $\frac{UT}{AT}=\frac{8}{12}=\frac{2}{3}$ *and* $\frac{BT}{ST}=\frac{6}{9}=\frac{2}{3}$ ❶ ΔBUT *is similar to* ΔSAT (*two corresponding sides are in the same ratio and the included angle is equal*)	*Many students started by stating that the ratios were equal (as if this was given), rather than showing that they were equal.* *i.e.* $\frac{UT}{AT}=\frac{BT}{ST}=\frac{2}{3}$. ***This was not given any marks.*** *Some students also made no mention of an angle, as if doing a SSS in ratio proof, but as one side is unknown, this is not possible. An angle is needed in the proof. (SAS in ratio)*
	(ii) $BU=10$ *as* ΔBUT *is a right* Δ ❶ $(6,8,10$ *pythag.triad*$)$ ❶ *OR* $\frac{BU}{15}=\frac{2}{3}\quad\therefore BU=\frac{30}{3}=10$ ❶ (*corresponding sides of similar* Δ*s are proportional*) ❶	*Generally well done.*
	End of Question 13	

Question 14 (15 marks)			Teacher's Comments
(a)		 ❶ diagram OR reason (see below **)**	
	(i)	$\angle ABC = 36° + 24°$ (*alternate* $\angle$*'s on parallel lines, adj.*sup.$\angle$*'s*) $= 60°$ ❶	
	(ii)	$d^2 = 35^2 + 48^2 - 2 \times 35 \times 48 \times \cos 60°$ ❶ $= 1849$ $d = \sqrt{1849}$ $= 43$ ❶ *the dis* tan *ce of AC is* 43*km*	
	(iii)	The bearing of A from C $= 360° - (\angle BCA + 24)$ ❶ $= 360 - (44°49' + 24°)$ $= 291°11'$ ❶	
(b)	**(i)**	$y = 3 + \cos 2x$ for $-\pi \le x \le \pi$  ❶ shape ❶ y-intercept ❶ endpoints	

	(ii)	$A=\int_0^{\frac{7\pi}{12}}(3+\cos 2x)\,dx$ $=\left[3x+\frac{\sin 2x}{2}\right]_0^{\frac{7\pi}{12}}$ ❶ $=\left[3\left(\frac{7\pi}{12}\right)+\frac{1}{2}\left(\sin\frac{7\pi}{6}\right)\right]-\left[3(0)+\frac{1}{2}(\sin 0)\right]$ $=\left[\left(\frac{7\pi}{4}\right)+\frac{1}{2}\left(\sin\frac{7\pi}{6}\right)\right]-[0]$ $=\frac{7\pi}{4}-\frac{1}{4}$ $=\frac{7\pi-1}{4}\quad units^2$ ❶	
(c)			
	(i)	P(A wins in one turn) = $\frac{2}{5}$ ❶	
	(ii)	B (wins on his first draw) $=\frac{3}{5}\times\frac{3}{5}$ ❶ $=\frac{9}{25}$	
	(ii)	P(A wins in fewer than 4 turns) = P(A wins in 1 turn) + P(A wins on 2nd turn) + P(A wins in 3 turns) $=\frac{2}{5}+\left(\frac{3}{5}\times\frac{2}{5}\times\frac{2}{5}\right)+\left(\frac{3}{5}\times\frac{2}{5}\times\frac{3}{5}\times\frac{2}{5}\times\frac{2}{5}\right)$ ❶ $=\frac{2}{5}+\frac{12}{125}+\frac{72}{3125}$ $=\frac{1622}{3125}$ ❶	
End of Question 14			

Question 15 (15 marks)			Teacher's Comments
(a)			
	(i)	$\frac{d}{dx}\left(x^2 \ln x\right)=x^2\frac{d}{dx}(\ln x)+\ln x\frac{d}{dx}\left(x^2\right)$ $=x^2\left(\frac{1}{x}\right)+(2x)\ln x$ $=x+2x\ln x$ ❶	*Generally well done.*
	(ii)	$\int 3x\left(1+\ln x^2\right)\,dx$ $=3\int x+x\ln x^2\,dx$ $=3\int x+2x\ln x\,dx$ ❶ $\left(\text{as} \ln x^n = n\ln x\right)$ $=3\left(x^2\ln x\right)+c$ ❶	*Many were unable to use the log laws and realise* $\ln x^2 \equiv 2\ln x$. *Many fudges in answer around this point.*
(b)			
	(i)	$f(x)=3-3x^2-x^3$ in the domain $-3\le x\le 2$ $f'(x)=-6x-3x^2$ $f''(x)=-6-6x$ $\therefore$ Stationary points occur when $f'(x)=0$ $f'(x)=-6x-3x^2$ *i.e.* $-6x-3x^2=0$ $-3x(2+x)=0$ $\therefore\ x=0, x=-2$ ❶ Nature of Stationary points: When $x=0,\ y=3$ and $f''(x)=-6-6(0)$ <0 (concave down) $\therefore (0,3)$ is a maximum turning point ❶ When $x=-2,\ y=3-3(-1)^2-(-2)^3=-1$ and $f''(x)=-6-6(-2)$ >0 (concave up) $\therefore(-2,-1)$ is a minimum turning point ❶	*Many did not take note the domain required, particularly for the sketch later.* *Some students are unclear on the following:* $f'(x)=0$ *always produces a stat. pt. You must always test for the* <u>*type*</u> *of stat pt.* $f''(x)=0$ *only produces a possible inflection point. You must always test for concavity change for a point of inflection.* *A few did not state points as required, but just gave* x and y *values. Otherwise, generally well done.*
	(ii)	Points of inflexion occur when $f''(x)=0$	

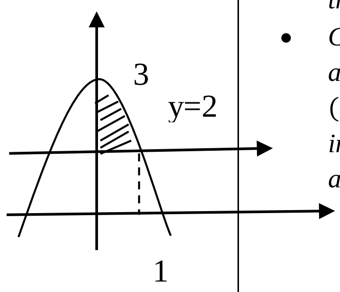
<table>
<tr><td></td><td></td><td>$f''(x) = -6 - 6x$
$i.e.\ -6 - 6x = 0$
$-6(1+x) = 0$
$\therefore\ x = -1$ and $y = 3 - (-1)^2 - (-1)^3$
$= 1$
possible point of inflexion at $(-1, 1)$ ❶

(ii)cont.
<table><tr><td>x</td><td>$x=-2$</td><td>$x=-1$</td><td>$x=0$</td></tr><tr><td>$f''(x)$</td><td>>0</td><td>$=0$</td><td><0</td></tr></table>
Sign/ concavity change either side
$\therefore$ point of inflexion at (-1, 1) ❶</td><td>Many did not test for inflection.</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>(iii)</td><td>

❶ shape & pt of inflexion
❶ y intercept & endpoints</td><td>End-points of domain frequently missed – these must be there when a domain is given!

A sketch needs to be in scale, be neat, in pencil and showing the points found. Many diagrams were very poor quality.</td></tr>
<tr><td></td><td>(iv)</td><td>Minimum value of the function is -17 ❶</td><td>Minimum value of a function is a number - <u>not</u> a point or a y-value!</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>(c)</td><td></td><td>A= $35000
n= 5x12=60 months
r = $\frac{18}{12}\% = 1 \cdot 5\% = 0 \cdot 015$ per month</td><td></td></tr>
<tr><td></td><td>(i)</td><td>Let A_n be the amount owing after n payments
M be the amount paid monthly/each instalment</td><td></td></tr>
</table>

		$\therefore A_1 = 35000 + 35000 \times 0\cdot015 - M$ $= 35000(1 + 0\cdot015) - M$ $A_2 = A_1 \times 1\cdot015 - M$ $= [35000(1\cdot015) - M] \times 0\cdot015 - M$ ❶ $= 35000 \times 1\cdot015^2 - M(1\cdot015) - M$ $= 35000 \times 1\cdot015^2 - M(1\cdot015 + 1)$	*Most successfully established the modification to A_1 to show A_2. Those who lost this mark did not make the link to A_1.*
	(ii)	Continued from part (i) $A_3 = A_2(1\cdot015) - M$ $= [35000(1\cdot015)^2 - M(1\cdot015 + 1)] \times 0\cdot015 - M$ $= 35000(1\cdot015)^3 - M[(1\cdot015)^2 + 1\cdot015] - M$ $= 35000(1\cdot015)^3 - M(1\cdot015^2 + 1\cdot015 + 1)$ $\vdots$ $A_{60} = A_{59}(0\cdot015) - M$ $= 35000 \times 1\cdot015^{60} - M(1\cdot015^{59} + 1\cdot015^{58} + 1\cdot015^{57} + \ldots + 1)$ ❶ (as a geometric series with $a = 1$, $r = 1\cdot015$, $n = 60$, gives) ❶ $= 35000 \times 1\cdot015^{60} - M\left(\frac{1 \times (1\cdot015^{60} - 1)}{1\cdot015 - 1}\right)$ $A_{60} = 0$ when loan is repaid after 60 months: $0 = 35000 \times 1\cdot015^{60} - M\left(\frac{1 \times (1\cdot015^{60} - 1)}{1\cdot015 - 1}\right)$ $M = \frac{(35000 \times 1\cdot015^{60}) \times 0\cdot015}{1\cdot015^{60} - 1}$ $= 888\cdot769$ $= \$889$ (to the nearest dollar) ❶	*Most were also less successful in establishing the continuing pattern and generalizing for A_n* *Many also did not make the link with a GP – make sure you state the values of a, r and n that apply.* *Most got the correct answer, although some poor calculator work was evident.*
		End of Question 15	

Question 16 (15 marks)			**Teacher's Comments**
(a)		$y=3-x^2 \;\therefore\; y^2=9-6x^2+x^4$ when $y=2,\ x=1$ ➊ $V=\pi\int_0^1\left(y^2\right)dx-\pi\int_0^1\left(2^2\right)dx$ $\therefore V=\pi\int_0^1\left(9-6x^2+x^4\right)dx-\pi\int_0^1(4)\,dx$ ➊ $=\pi\left[\left(9x-\frac{6x^3}{3}+\frac{x^5}{5}\right)-(4x)\right]_0^1$ ➊ $=\pi\left[\left(9(1)-2(1)^3+\frac{(1)^5}{5}\right)-4(1)-[0]\right]$ $=\pi\left[\frac{36-20}{5}\right]$ $=\frac{16\pi}{5}u^3$ ➊  	• *Not answered very well* • *Students calculated the area bounded by the x=axis, the curve, the y-axis and the line y=2 and did not realise that the question did not refer to the x-axis as a boundary. This led to them calculating the area under the line y=2 and using incorrect x limits of 0 and* $\sqrt{3}$ • *A few students rotated the curve about the y-axis instead of the x-axis* • *There were quite a few careless errors such as forgetting to subtract area under the line when calculating the volume.* • *Conceptual errors were also apparent with some students using* $(3-x^2-2)^2$ *in their calculations, indicating the need to improve algebraic skills*
(b)	**(i)**	$P=Ae^{-kt}$ $\frac{dP}{dt}=-k\,Ae^{-kt}$ $=-k\,P \quad (as\ P=Ae^{-kt})$ $\therefore\ P=Ae^{-kt}$ is a solution ➊	• *Generally answered well* • *A few students tried to use integration but were unable to arrive at the correct answer*

	(ii)	$Given\, t = 0, P = 2000 \; and \; t = 300, P = 1000$ $\therefore when\, t = 0, P = Ae^{-kt}$ $2000 = Ae^{0}$ $A = 2000$ $t = 300, \; P = 1000$ $1000 = 2000\left(e^{-k(300)}\right)$ $\ln\frac{1}{2} = -300k$ $\therefore k = \frac{\ln\frac{1}{2}}{-300}$ $= \frac{\ln 2}{300} \; (since \ln\frac{1}{2} = \ln 2^{-1} = -\ln 2)$ $= 0\cdot 00231$ ❶ $When\, t = 400, P = 2000e^{-k(400)}$ $= 793\cdot 7$ ≈ 794 ❶ *After 400 days the population of the species is 794 (nearest whole number)*	• *Generally answered well* • *Students should round up in their final answer and use the exact value of k in subsequent calculations*
	(iii)	$400 = 2000e^{-kt}$ ❶ $\ln\frac{400}{2000} = \ln e^{-kt}$ $\ln\frac{1}{5} = -kt$ $t = \frac{\ln\frac{1}{5}}{-k}$ $= 696\cdot 6$ $\therefore$ *after 697 days population of the species will drop below 400* ❶	• *Generally answered well* • *Students were awarded marks if correct calculations performed with a carry on error*

(c)	**(i)**	$Area = \frac{1}{2}r^2\theta$ $8 = \frac{1}{2}r^2\theta$ $\frac{16}{\theta} = r^2$ $r = \sqrt{\frac{16}{\theta}}\ , \quad r > 0$ $= \frac{4}{\sqrt{\theta}} \times \frac{\sqrt{\theta}}{\sqrt{\theta}}$ $= \frac{4\sqrt{\theta}}{\theta}$ ❶	• *Generally answered well*
	(ii)	$P = l_{BC} + 2r$ $= r\theta + 2\left(\frac{4}{\sqrt{\theta}}\right)$ $= \left(\frac{4}{\sqrt{\theta}}\right) \times \theta + 2\left(\frac{4}{\sqrt{\theta}}\right)$ $= \left(\frac{4\sqrt{\theta}}{\theta}\right) \times \theta + \frac{8}{\sqrt{\theta}}$ ❶ $P = \frac{8}{\sqrt{\theta}} + 4\sqrt{\theta}$	• *Generally answered well*
	(iii)	$P = \frac{8}{\sqrt{\theta}} + 4\sqrt{\theta} = 8\theta^{\frac{-1}{2}} + 4\theta^{\frac{1}{2}}$ $\therefore P' = -4\theta^{\frac{-3}{2}} + 2\theta^{\frac{-1}{2}}$ $= \frac{-4}{\sqrt{\theta^3}} + \frac{2}{\sqrt{\theta}}$ $= \frac{-4}{\theta\sqrt{\theta}} + \frac{2}{\sqrt{\theta}}$ ❶ $P'' = -4\left(\frac{-3}{2}\right)\theta^{\frac{-5}{2}} - \theta^{\frac{-3}{2}}$ $= 6\theta^{\frac{-5}{2}} - \theta^{\frac{-3}{2}}$ $= \frac{6}{\theta^2\sqrt{\theta}} - \frac{1}{\theta\sqrt{\theta}}$ ❶	• *Full marks were awarded if students correctly used the first derivative to determine maxima/ minima instead of the second derivative. However, if they did not show values for the first derivative around the stationary point marks were deducted.* • *A few students used the quotient rule to differentiation instead of converting each term to index form before differentiating – this sometimes led to careless errors* • *Marks were deducted if a check for maxima/ minima was not carried out.* • *A few students unnecessarily calculated the perimeter once the value of θ was determined*

		(iii) cont. $Stationary\ values: when\ P' = 0$ $P' = \frac{-4}{\theta\sqrt{\theta}} + \frac{2}{\sqrt{\theta}} = 0$ $\frac{-4+2\theta}{\theta\sqrt{\theta}} = 0$ $2\theta = 4$ $\theta = 2$ ❶ $Nature\ of\ stationary\ value:$ $When\ \theta = 2$ $P'' = \frac{6}{\theta^2\sqrt{\theta}} - \frac{1}{\theta\sqrt{\theta}}$ $= \frac{6}{4\sqrt{2}} - \frac{1}{2\sqrt{2}}$ $= \frac{4}{4\sqrt{2}}$ > 0 ❶ $\therefore$ Minimum perimeter when θ=2	

Mathematics: Multiple Choice Answer Sheet

1	A
2	C
3	D
4	D
5	C
6	C
7	A
8	A
9	D
10	B